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MARKOV
The New Road to WAR

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2018-08-10

SALO 2.0: Bayesian ordered logit for SoG and penalties

$$\Pr(\text{away shot}) = \Lambda(\alpha_{sa} - X\beta_s - Z\gamma_s)$$
$$\Pr(\text{home shot}) = \Lambda(\alpha_{sh} + X\beta_s + Z\gamma_s)$$

(likelihood)

$$\Pr(\text{away pen}) = \Lambda(\alpha_{pa} - X\beta_p - Z\gamma_p)$$
$$\Pr(\text{home pen}) = \Lambda(\alpha_{ph} + X\beta_p + Z\gamma_p)$$

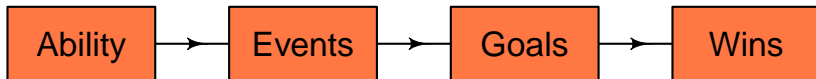$$\gamma_s \sim N(0, \sigma_s) \qquad \gamma_p \sim N(0, \sigma_p) \qquad \text{(prior)}$$

$$GP \sim \text{BB}(82, \alpha_G + \gamma_s\beta_G, \phi) \qquad \text{([name?])}$$

WAR: combining multiple abilities into an overall value

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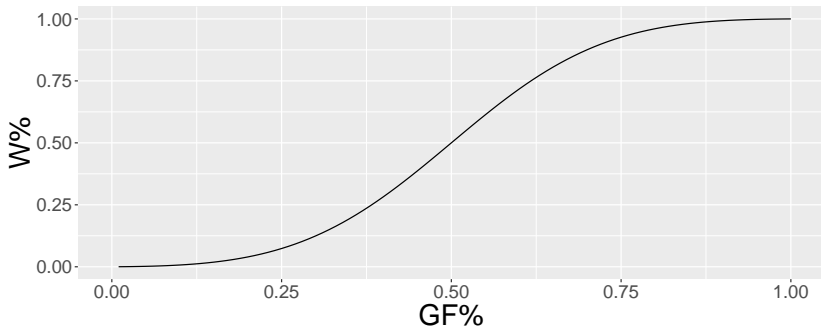○○

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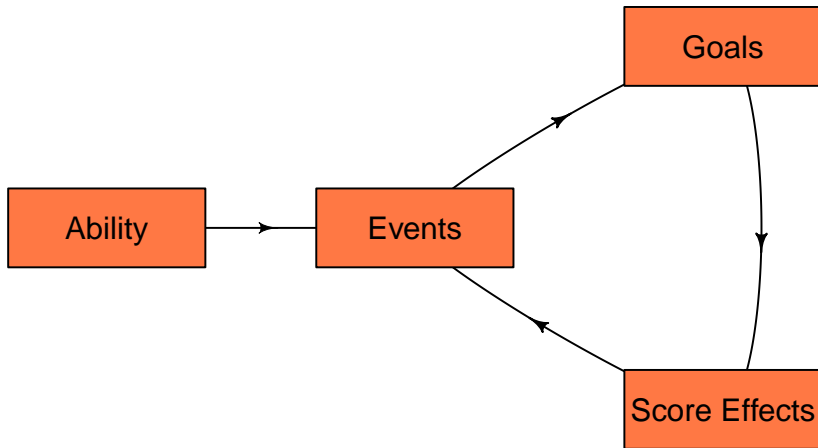○○○○

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cWAR linearly maps micro ability to macro results

Wins (added) aren't linear in goals (added)

```
G <- (0:100) / 100
W <- skellam::pskellam(-1, (1 - G) * 6, G * 6)
W <- W / (1 - skellam::dskellam(0, (1 - G) * 6, G * 6))
```
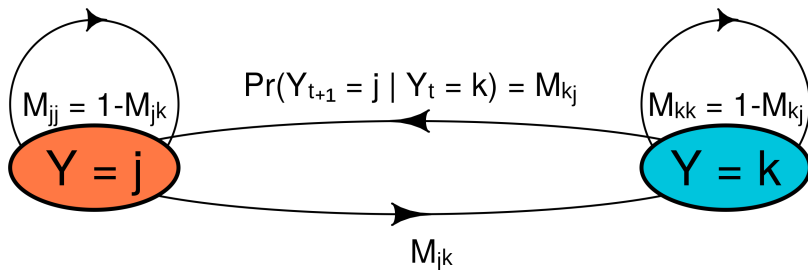

▶ bWAR 2.x accounts for this

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Goals (added) aren't (globally) linear in ability, either!

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A *Markov chain* randomly changes state in discrete time according to a matrix of transition probabilities

$$M = \begin{bmatrix} M_{jj} & M_{jk} \\ M_{kj} & M_{kk} \end{bmatrix}$$

$M_{jj} = 1\text{-}M_{jk}$

$Pr(Y_{t+1} = j \mid Y_t = k) = M_{kj}$

$M_{kk} = 1\text{-}M_{kj}$

Y = j

Y = k

M_{jk}

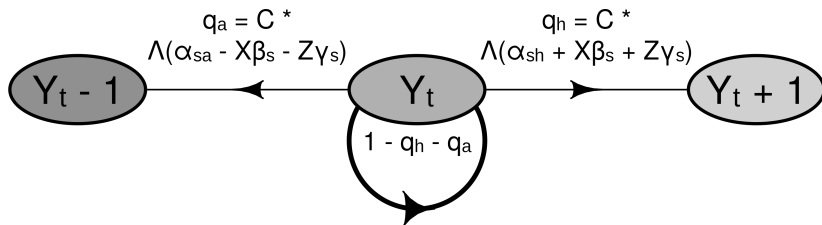*Markov property*: Distribution of Y_t given Y_0 equals M^t

SALO defines a game state space and transition matrix
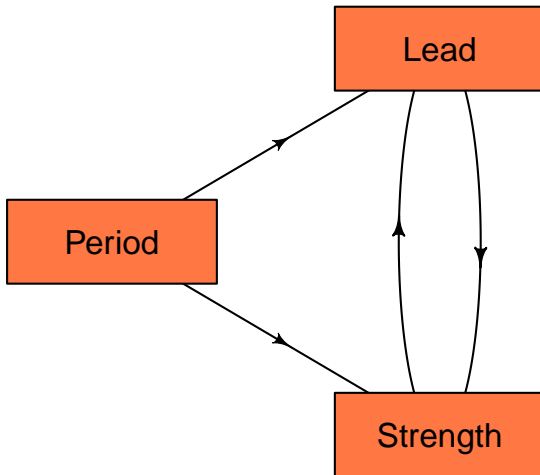
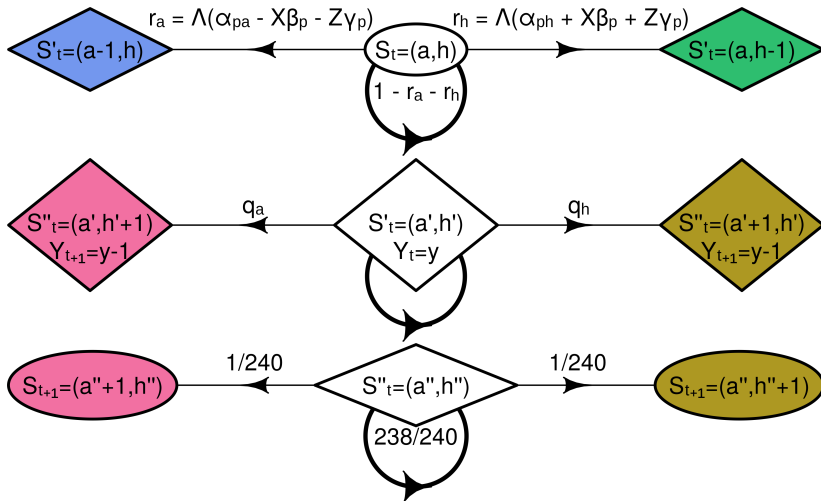# Transition matrices defined separately for each period

$$M^* = (M_1)^{2400} \times (M_2)^{2400} \times (M_3)^{2400}$$

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Lead change prob. is SALO shot prob. × (constant) Sh%

$q_a = C *$
$\Lambda(\alpha_{sa} - X\beta_s - Z\gamma_s)$

$q_h = C *$
$\Lambda(\alpha_{sh} + X\beta_s + Z\gamma_s)$

$Y_t - 1$

Y_t

$Y_t + 1$

$1 - q_h - q_a$

Penalties, PPG, and expiration cause strength changes

$S'_t=(a-1,h)$ $r_a = \Lambda(\alpha_{pa} - X\beta_p - Z\gamma_p)$ $S_t=(a,h)$ $r_h = \Lambda(\alpha_{ph} + X\beta_p + Z\gamma_p)$ $S'_t=(a,h-1)$

$1 - r_a - r_h$

$S''_t=(a',h'+1)$ $Y_{t+1}=y-1$ q_a $S'_t=(a',h')$ $Y_t=y$ q_h $S''_t=(a'+1,h')$ $Y_{t+1}=y-1$

$S_{t+1}=(a''+1,h'')$ $1/240$ $S''_t=(a'',h'')$ $1/240$ $S_{t+1}=(a'',h''+1)$

$238/240$

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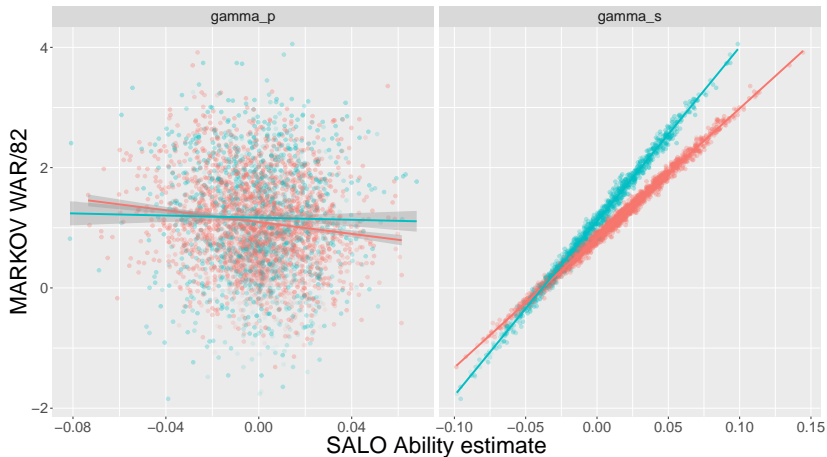MARKOV
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Assumptions are added for finiteness and completeness

- ▶ M is cropped to 135×135 (15 leads, 9 strength states)
- ▶ Average ice time: forwards 15:00, defensemen 20:00
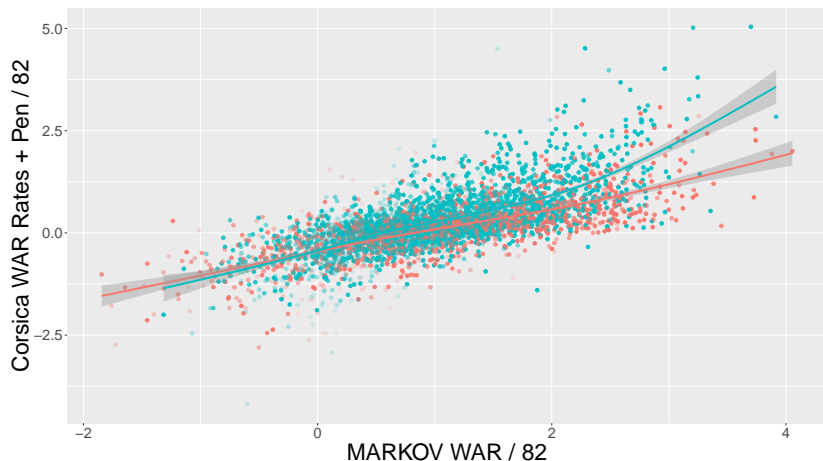- ▶ Model-based replacement player

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Top players mostly D (ice time assumption is severe)

season	player	pos	mean	sd
20152016	KRIS LETANG	D	4.055	1.550
20152016	PAVEL DATSYUK	C	3.916	1.267
20132014	MARK GIORDANO	D	3.880	1.572
20152016	JAKE MUZZIN	D	3.741	1.582
20152016	HAMPUS LINDHOLM	D	3.740	1.573
20152016	COLTON PARAYKO	D	3.727	1.574
20142015	PATRICE BERGERON	C	3.701	1.258
20172018	RADKO GUDAS	D	3.449	1.602
20152016	OLIVER EKMAN-LARSSON	D	3.382	1.547
20152016	CARL HAGELIN	L	3.356	1.248

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WAR is almost linear in shots and unrelated to penalties

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Corsica WAR is not quite linear in MARKOV WAR

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https://www.salohockey.net